Combustion Instability in Solid Propellant Rockets

The following 6 papers on Nonlinear Acoustic Instability and 3 papers on Nonacoustic Instability were presented at the AIAA Solid Propellant Rocket Conference in Palo Alto, California, January 29–31, 1964. Papers on Linear Acoustic Instability were published in the June 1964 AIAA Journal.

Nonlinear Effects in Instability of Solid-Propellant Rocket Motors

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This paper is in two parts. The first section discusses briefly the various kinds of nonlinear effects associated with instability in solid-propellant rockets and attempts to provide brief surveys of the ways in which various nonlinear phenomena may manifest themselves. The second section reviews and extends some of the work that has been carried out at the Applied Physics Laboratory on nonlinear instability phenomena arising from erosion. This work has been concerned with effects of erosion on stability and with the waveform of finite amplitude oscillations in motors with cylindrical propellant charges. In this connection, it is found that, especially for axial modes, significant waveform distortion may develop even for relatively small amplitudes of the pressure oscillation and that the wave shape tends to be a rather sensitive function of the time dependent response of the propellant.

Nomenclature

A = dimensionless pressure amplitude
a = cavity radius
c = no-flow sound speed $(\gamma\bar{P}/\bar{\rho})$
F_N = defined in text following Eq. (5c)
Im = imaginary part of
$K(\omega)$ = erosive response function at angular frequency ω
L = cavity length (choked end nozzle burner); or cavity half length (T-burner)
M_b = Mach number of normal unperturbed velocity component of the burnt gases leaving the burning zone
M_* = Mach number of unperturbed velocity component normal to exhaust port plane
$\bar{P}$ = unperturbed pressure
p = acoustic pressure
Re = real part of
dS = differential element of cavity surface area (outward-directed normal)
t = time
$\mathbf{u}$ = acoustic velocity
$\bar{\mathbf{v}}$ = unperturbed flow velocity
v_e = erosive velocity tangential to propellant surface
Y_* = acoustic admittance at exhaust port plane
y_* = dimensionless ("reduced") acoustic admittance at exhaust port plane
z = space coordinate in axial direction
α_l = linear acoustic field decay constant
γ = specific heat ratio
$\tilde{\epsilon}$ = dimensionless Fourier pressure, $p = Re(\bar{P}\tilde{\epsilon}e^{i\omega t})$
$\tilde{\mu}_p$ = dimensionless Fourier mass flux due to acoustic pressure response
ξ = defined by Eqs. (4, 5a, 5b, 5c)
$\bar{\rho}$ = unperturbed density
ω = angular frequency
Ω = angular frequency of dominant acoustic mode
$\langle\ \rangle$ = time average
$\overline{(\ )}$ = unperturbed quantity
$\tilde{(\ )}$ = Fourier component, i.e., $F = Re(\tilde{F}e^{i\omega t})$
$(\ )_*$ = exhaust port plane
$(\ )_a$ = conditions at $r = a$
$(\ )_L$ = conditions at $z = L$

Presented as Preprint 64-140 at the Acoustic Combustion Instability session (cosponsored by the Department of Defense Technical Panel on Solid Propellant Instability of Combustion) at the AIAA Solid Propellant Rocket Conference, Palo Alto, Calif., January 29–31, 1964; revision received March 16, 1964. This work was supported by the Bureau of Naval Weapons, Department of Navy, under Contract No. NOw62-0604-c.

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I. Introduction

EVEN a stable rocket motor is typically a very nonlinear system, and thus the term nonlinear may refer to a variety of its aspects. Here, however, we are concerned chiefly with nonlinearities associated with periodic oscillations within the motor. Ordinarily, one expects that, when the fluctuations are sufficiently small, their second- and higher-order powers may be neglected, and linear considerations pertain. But, when the departure from steady-state behavior is sufficiently great, this neglect is no longer permissible, and the nonlinear character of the time dependent behavior of the motor would become evident. It should be noted, however, that under some circumstances the behavior can be nonlinear even for small fluctuations. In this connection, we recall that the effect of an erosive velocity on the burning rate of a propellant does not depend on the direction of that velocity, but that it depends on its absolute magnitude. Thus, in a region where the mean erosive velocity vanishes, the response of the propellant to the fluctuating erosive velocity is nonlinear even for infinitesimal fluctuations.

We shall recognize four ways in which nonlinear phenomena associated with periodic oscillations manifest themselves. Most obvious is the fact that unstable rockets exhibit oscillations that grow to a limiting amplitude. This limiting action clearly requires that some nonlinear effect has appeared. Another clear indication is the appearance of waveforms, which are nonsinusoidal in time, which are often observed at high amplitudes and are particularly common for axial modes in cylindrical propellant channels. Changes in the mean ballistic properties during oscillatory combustion are another manifestation commonly recognized. These arise from changes in the mean burning rate of the propellant when subjected to fluctuating pressures and velocities parallel to its surface, from interference with the combustion efficiency, and from the introduction of new flow fields (triggered by acoustic streaming) which affect both the burning rate and the flow through the nozzle. Finally, nonlinear instability itself may be observed. In this case a motor may be stable to small perturbations (linearly stable) but may go into sustained oscillation when a sufficiently large perturbation is imposed. The work of Dickinson and Brownlee[1] beautifully illustrates such a phenomenon for axial modes. The work of Swithenbank and Sotter[2] illustrates nonlinear instability for transverse modes.

It will be recalled that the balance of acoustic gains and losses is of fundamental importance to both linear and nonlinear combustion instability. Thus, in general, if the acoustic gains exceed the acoustic losses, the amplitudes of the fluctuations will tend to grow. If the motor retains its integrity, the losses will ultimately grow faster with amplitude than the gains, so that the oscillations will become limited at some finite value. These gains and losses will include, for example, the response of the burning propellant and the various kinds of gains or losses which may occur at the orifice and other boundaries of the motor cavity, as well as the various kinds of effects associated with the cavity volume.

These considerations illuminate the general nature of the difficulties facing the purely empirical approach. The limiting amplitude, for example, is determined by the joint effect of gains and losses. Thus, measurement of limiting amplitude does not illuminate individually either the gains or the losses, but only their composite effect. Only when a single gain and a single loss mechanism are dominant can one really expect to obtain useful scaling laws. But, in general, the relative importance of the various mechanisms will vary with scale (as they do in the linear case), so that it is not surprising to find that empirical study of rocket motors has met with a limited degree of success when applied to the nonlinear problem similar to that encountered when applied to the problem of linear stability. Just as in the linear stability case, it seems quite clear that if we wish to be able to predict the unstable behavior of a proposed motor, or even to be able to vary with confidence the design of a given motor in order to reduce instability, it will be important to study the various gain and loss mechanisms individually.

An attempt has been made to consider the nonlinear properties of the combustion process from a general analytical point of view. Starting from the basic molecular composition of the propellant, an attempt was made to solve for the nonlinear behavior by applying the basic laws of chemistry and fluid dynamics. Unfortunately, however, the complexity of the calculation seems to make it impractical even for the fastest computing machines of today.[3] In view of the well-known mathematical difficulties associated with even steady-state burning, it is perhaps not surprising that this approach also has not met with success.

In recognition of the seriousness of the difficulties that are inherent in the problems of nonlinear combustion instability, most theoretical and experimental work tends to deal primarily with one or a selected few aspects of the over-all problem. Some progress has been made in the low-amplitude (but nonlinear) regime in connection with erosive response.[4] Some success has also been achieved in the high-amplitude domain with the aid of shock wave theory.[5]

A phenomenon of major importance, with dramatic effects on performance, is brought out by Swithenbank and Sotter[2] and by Flandro,[6] who deal with the generation of vortex flow by intense sound waves in rocket motors.

It has already been noted that there will be nonlinear effects associated with each of the several acoustic gain and loss mechanisms if the amplitude of the disturbance is "large." Of course, each mechanism has its own criteria for "large." For example, since the erosive response of a propellant depends on the magnitude (but not on the direction of the erosive velocity), the erosive response will become nonlinear for acoustic velocities large enough (compared with the mean flow velocity) to produce flow reversal. If we consider an inside burning cylindrical grain, for example, the Mach number at the midpoint of the propellant channel might be, say, 0.02. Thus, a pressure amplitude in the fundamental axial mode of only about 2% of the mean pressure would be sufficient to cause flow reversal and nonlinear effects. The burnt gas, on the other hand, which may be regarded as an approximately linear acoustic medium for such small Mach numbers, will not be expected to call attention to its nonlinear properties for disturbances whose associated Mach numbers are less than perhaps 10^{-1}. Under these conditions, it is reasonable to approach the nonlinear problem by an extension of the acoustic treatment, and to this end we devote the rest of this paper.

II. Some Nonlinear Effects of Erosion

In the following, we shall confine our attention to the erosive response of propellants to finite amplitude disturbances which are not so large that the nonlinear properties of the acoustic medium (burnt gas) and the pressure response of the propellant need be considered. We shall consider, in particular, effects that erosion may have on nonlinear stability, on limitation of amplitude, and on waveform for the case of inside burning cylindrical charges. The first two of these effects have also been considered in a somewhat more crude fashion by three of the present authors.[4] Although the qualitative nature of the earlier results remains valid, certain quantitative modifications should be made as the slightly more elaborate development that follows will show.

Flow Effects in Nonlinear Erosive Instability of Axial Modes

As expected, the basic analytical problem concerns the formulation of the acoustic gain-loss balance. It is shown in Ref. 7 that, to second order in the fluctuating quantities (and to first order in mean flow Mach number), the criterion for neutral stability is given by

$$0 = \left\langle \int_S d\mathbf{S} \cdot \left[p\mathbf{u} + \frac{p^2\bar{\mathbf{v}}}{\bar{\rho}c^2} + \bar{\rho}(\bar{\mathbf{v}} \cdot \mathbf{u})\mathbf{u} \right] \right\rangle \tag{1}$$

where the $\langle \ \rangle$ indicate the time average, p is the acoustic pressure, $\mathbf{u}$ is the acoustic velocity, $\bar{\mathbf{v}}$ is the unperturbed velocity in the absence of acoustic disturbance, c is the unperturbed no-flow sound speed, $\bar{\rho}$ is the unperturbed density, and S is the surface bounding the cavity. Equation (1) is particularly useful because it is necessary to evaluate the fluctuating quantities only to first order to obtain the stability criterion correct to second order (because only products of the fluctuating quantities occur). The first term of Eq. (1) is the mechanical work term, which is familiar from no-flow acoustics. The remaining terms arise because of the mean flow. In the previous work,[4] the criterion for stability was developed in terms of the no-flow criterion, and, thus, it included only the mechanical work term of Eq. (1). Further, the boundary conditions at the port plane were assumed to correspond either to a pressure node or a velocity node. The present development makes explicit provision for the case of a choked end nozzle and for the case of nonchoked center orifice (T-burner).

It is perhaps obvious (see Ref. 7) that it will be permissible for the present problem to use the no-flow-no-loss-linear-approximation to the pressure field and the tangential component of the velocity field in the evaluation of Eq. (1) over the boundaries of the propellant channel. The normal component of fluctuating velocity at the boundary is to be expressed as a product of the fluctuating pressure and an appropriate acoustic admittance. We may assume that the admittances at the port plane and at the propellant surface are very small, so that the appropriate no-flow-no-loss approximate acoustic fields are

$$p = \bar{P}A \cos(N\pi z/2L) \cos\omega t \tag{2a}$$

$$\mathbf{u} = \mathbf{1}_z(cA/\gamma) \sin(N\pi z/2L) \sin\omega t \qquad N = 1,2,3,4\ldots \tag{2b}$$

where N denotes the particular mode under consideration, A is the dimensionless pressure amplitude, $\mathbf{1}_z$ is a unit vector in the axial direction, ω is the angular frequency, and L is the cavity length for the choked end nozzle burner and the cavity half-length for the T-burner. (Note that $z = 0$ corresponds to the head end of the former burner and the left end wall of the latter.) Following Ref. 4, the boundary condition at the propellant surface is related to the acoustic pressure response function $(\tilde{\mu}_p/\bar{\epsilon})$ and the acoustic erosive response function K. Thus, the boundary condition at the propellant surface where $r = a$ is (in vector notation)

$$d\mathbf{S}\cdot\mathbf{u}_a = -2\pi a\, \dot{c} M_b dz \mathrm{Re}\left\{\left[\frac{\tilde{p}}{\bar{P}}\left(\frac{\tilde{\mu}_p}{\bar{\epsilon}} - \frac{1}{\gamma}\right) + K(\omega)\left[\widetilde{|v_e| - \langle|v_e|\rangle}\right]\right] e^{it\omega}\right\} \tag{3a}$$

where M_b is the Mach number of the burnt gas leaving the burning zone, Re denotes real part of, v_e is the erosive velocity, i.e., the component of velocity tangent to the propellant surface, and, as usual, the tilda denotes the Fourier amplitude component. Furthermore, from Ref. 4, the erosive velocity is approximated by

$$v_e = \bar{v}_e + \frac{cA}{\gamma} \sin\left(\frac{N\pi z}{2L}\right) \sin(\omega t) \tag{3b}$$

where $\bar{v}_e = |v_L|z/L$ for the choked end nozzle cavity and for $z < L$ in the T-burner and $\bar{v}_e = |v_L|(z - 2L)/L$ for $L < z \leq 2L$ in the T-burner. Then,

$$[\widetilde{|v_e| - \langle|v_e|\rangle}] = -\frac{icA}{\pi\gamma} \sin\left(\frac{N\pi z}{2L}\right)\left[\pi - 2\sigma + \sin 2\sigma\right] \mathrm{signum}(\bar{v}_e) \tag{3c}$$

where

$$\cos\sigma = \frac{\gamma|\bar{v}_e|}{cA|\sin(N\pi z/2L)|} \qquad 0 \leq \sigma \leq \frac{\pi}{2} \tag{3d}$$

$$\sigma = 0 \quad \text{if} \quad \bar{v}_e\gamma > cA\left|\sin\left(\frac{N\pi z}{2L}\right)\right| \tag{3e}$$

When the foregoing equations are substituted into Eq. (1), one obtains for the neutral stability criterion

$$0 = \left\{\mathrm{Re}\left[\frac{\tilde{\mu}_p}{\bar{\epsilon}} - \xi_N\right] + \frac{2c}{\gamma}\,\mathrm{Im}(K)F_N\left(\frac{2\gamma M_*}{A}\right)\right\} \tag{4}$$

where Im denotes the imaginary part. In general, the quantity ξ_N depends on the temperature response of the burning propellant, as well as on the acoustic mode and the cavity geometry. However, previous calculations suggest that little error will be introduced by assuming that the temperature variations of the gas leaving the burning surface obey the isentropic relationship, and we are then led to the rather simple results given below. [If an isothermal, instead of isentropic, relationship were assumed, $(\gamma - 1)/\gamma$ must be added to each ξ_N given below.]

Choked End Nozzle Cavity

$$\xi_N = (2/\gamma) + 2y_* \qquad N \text{ even}^{\P} \tag{5a}$$

T-Burner

$$\xi_N = 0 \qquad N \text{ odd} \tag{5b}$$

$$\xi_N = (2/\gamma) + 2y_* \qquad N \text{ even} \tag{5c}$$

where y_* is the dimensionless exhaust port plane admittance defined in terms of the actual specific acoustic admittance Y_* by $y_* = Y_*(\bar{P}/M_*c_*)$. The function $F_N(2\gamma M_*/A)$ vanishes for N even and equals $1/N\pi$ for N odd for amplitudes such that $A < (2\gamma M_*/\pi N)$ and is plotted for larger A in Ref. 4 for two values of N.

In the absence of wall and gas volume losses and in the absence of possible nonlinear effects other than the erosive response of the propellant, the satisfaction of Eq. (4) implies neutral stability. If the equation is not satisfied for some mode N and if the term on the right-hand side is positive in that mode, instability is indicated. We note that the result expressed by Eq. (4) differs from the result of Ref. 4, as expressed there by Eq. (9a), where the term $[(\tilde{\mu}_p/\bar{\epsilon}) - (1/\gamma)]$ appears instead of the term $[(\tilde{\mu}_p/\bar{\epsilon}) - \xi_N]$, which we see in Eq. (4). Consequently, in the results and discussion of Ref. 4, whenever one sees the quantity $[(\tilde{\mu}_p/\bar{\epsilon}) - (1/\gamma)]$, one should replace it by $[(\tilde{\mu}_p/\bar{\epsilon}) - \xi_N]$. The results of Ref. 4 are found to be otherwise valid.

In brief summary, we will recall that erosion may either enhance stability, inhibit it, or leave it unchanged. What the effect will be depends on the geometry, the mode, and the erosive response function of the propellant and may be calculated (for not too large amplitudes) by the method just developed. Detailed calculations presented here pertain to the case of axial modes of inside burning propellant channels with approximately rigid boundary conditions. For this case, the influence on the stability criterion of both the erosive response and the pressure response of propellants is given by Eq. (4). It will be noted that the effect of erosion will be stabilizing or destabilizing according to whether the sign of $F_N Im[K(\omega)]$ is negative or positive. In the choked end nozzle burner, the erosive response of the propellant makes no contribution to stability for amplitudes (A), which are small enough so that flow reversal does not take place anywhere within the channel since F_N vanishes for $A < (2\gamma M_*/\pi N)$, $N = 2,4,6,8,\ldots$. If we consider the fundamental axial mode $(N = 2)$, we find that $F_2 \leq 0$, so that the effect of erosion will be destabilizing if the imaginary part of the erosive response function is less than zero and if the amplitude should exceed the critical value for flow reversal. A more detailed discussion will be found in Ref. 4.

Harmonic Generation and Waveform

It is clear that, whenever flow reversal occurs, erosive response of propellants will lead to the generation of harmonics. Thus, the pressure waveform, for example, often may be expected to depart from sinusoidal even at quite low amplitudes whenever the geometry is such that flow reversal can occur at low amplitude. Thus it is quite possible, for example, that steep fronted waveforms, somewhat resembling shock fronts, may be generated.

In order to investigate the effect of erosion on waveform, the derivation of Eq. (4) may be generalized to the case where more than one mode is excited.[8] It is assumed that the amplitudes of the harmonics are small compared with the amplitude of the fundamental. Attention is further restricted to conditions for which the motor is linearly stable at the frequencies of the harmonics, that is, the harmonic frequencies

¶ For the admittance to be defined, N odd does not lie within the realm of our approximation.

exist only because of the response of the propellant to the erosive velocity of the fundamental. It turns out, as one would expect, that the amplitude of a harmonic frequency may be related to the corresponding value of the linear decay rate constant characteristic of the cavity for that frequency and to the quantities characterizing the erosive response, such as the erosive response function $K(\omega)$.

The analysis of Ref. 8 shows, for example, that the pressure at the head end of the motor (of the geometry under consideration here) may be expressed in the form

$$\frac{P - \bar{P}}{\bar{P}} = A \cos\Omega t + \sum_{\nu=2}^{\infty} \left\{ \frac{K(\nu\Omega)}{\alpha_l(\nu\Omega)} G_\nu \cos\left[\nu\Omega t + \frac{\pi\nu}{2} + \arg K(\nu\Omega)\right]\right\} \quad (6)$$

where Ω is the fundamental frequency, α_l is the linear decay rate constant for the motor at the frequency $\nu\Omega$, and G_ν is a rather complicated function (a generalization of F_N) of the amplitude A of the fundamental, no-flow sound speed c, specific heat ratio γ, mean flow field, and motor geometry.

Figure 1 illustrates several waveforms that have been obtained using various assumed values of the relevant quantities as shown below:

$$cK(\nu\Omega) = 2 \qquad \alpha_l(\nu\Omega) = -25 \text{ sec}^{-1}$$
$$c = 10^5 \text{ cm/sec} \qquad L = 25 \text{ cm}$$
$$M_* = 0.1 \qquad \gamma = \tfrac{5}{4}$$

and for the two values of $\arg K(\nu\Omega)$ indicated on the two curves. We note especially the steep fronted wave front that is shown in Fig. 1. The approximations of the present treatment limit the steepness that may be obtained, and one expects that an analysis, which would be valid for higher amplitude harmonics, could yield very steep wave fronts. We would like to emphasize again that the nonlinear behavior shown in these figures does not result from the usual mechanisms of shock formation, since we have treated the gas as purely elastic in the usual sense of linear acoustics. The cause of the distorted waveforms being considered here lies entirely in the fact that the propellant responds to an erosive velocity without regard to the direction of that velocity.

III. Concluding Remarks

It seems clear that even for quite small amplitudes we may sometimes expect to observe appreciable nonlinear effects arising from the erosive response of propellants. The analysis indicates that under some conditions acoustic erosion may contribute toward nonlinear instability, to limiting the amplitude of oscillations, and to waveform distortion. What effects will be observed depends on the pressure response function and the erosive response function, as well as on quantities further specifying the acoustic properties of the motor. Clearly, so long as the propellant response functions remain unknown, the analysis can not be applied in detail to any real problem. Of course, if we look at the problem the other way around, so to speak, it is not inconceivable that careful experiments on both the linear and the nonlinear behavior may give substantial information about the erosive response. This is a point that Sirignano and Crocco[5] emphasize in regard to their formulation of nonlinear effects that arise (hopefully only at larger amplitudes than those considered by us) because of the nonlinear behavior of the burnt gas as an acoustic medium.

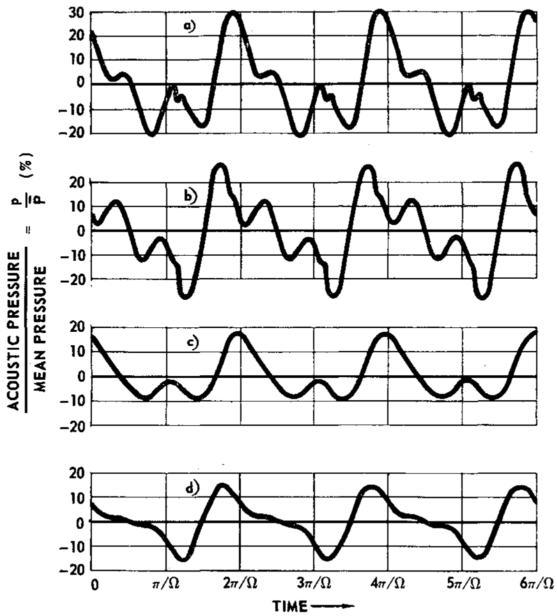

Fig. 1 Waveform distortion due to nonlinear erosion in the first axial modes of a) center nozzle motors, A = 16%, arg$K$ = 0°; b) center nozzle motors, A = 16%, arg$K$ = 90°; c) choked end nozzle motors, A = 10%, arg$K$ = 0°; and d) choked end nozzle motors, A = 10%, arg$K$ = 90°. Other parameter values are given in text.

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